Math 340 - Final Exam - December 20, 2022

Dr. Ebrahimian

Time: 120 minutes

Calculators and notes are not allowed. Completely and clearly justify your answers. Solutions to different problems should go on separate pages and in order.

1. (10 pts) Evaluate the determinant of the following 3×3 matrix using row operations. You <u>must</u> use row operations.
$$\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$$

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2. (10 pts) Determine if the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x + 2y, xy)$ is linear.

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3. (10 pts) Using $\epsilon - \delta$ definition of limit, prove $\lim\limits_{(x,y)\to(1,0)} xy = 0$.

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4. (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Define $g(x, y) = f\left(\frac{x+y}{x-y}\right)$ for every $x \neq y$. Prove that
$$xg_x + yg_y = 0.$$

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5. Let $S = \{(x_1, \ldots, x_n) \in \mathbb{R}^n \mid x_1^2 + x_2^4 + \cdots + x_n^{2n} = 1\}$.
   (a) (5 pts) Determine if S is compact.
   (b) (5 pts) Determine if S is open.

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6. (10 pts) Consider the quadratic form $Q(x, y, z) = x^2 + 4y^2 + 3z^2 - 2xy - 4yz - xz$. Write down the matrix of this quadratic form and use this matrix to determine if the origin is a local minimum, a local maximum or a saddle point for this quadratic form.

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7. (10 pts) Consider the parallelogram region R whose vertices are $(1, 1), (2, 2), (5, 2), (6, 3)$. Evaluate
$$\iint_R (y - x)^2 \ln(4y - x)\, dA.$$

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8. (10 pts) Find the volume of the solid region in $\mathbb{R}^3$ that lies inside the sphere $x^2 + y^2 + z^2 = 2z$ and above the cone $z = \sqrt{x^2 + y^2}$.

More Problems on the Back!

9. (10 pts) Use the Stokes' Theorem to find the work done by the vector field $\mathbf{F} = (e^x + z)\mathbf{i} + xz\mathbf{j} + \mathbf{k}$ on a particle that moves along the line segments from $(0, 0, 0)$ to $(1, 1, 1)$, then to $(0, 0, 2)$, and then back to the origin. Stop when you get an iterated double integral!

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10. (10 pts) Let E be a solid region in $\mathbb{R}^3$ whose boundary Σ is smooth and is oriented outward from E. Prove that the following surface integral only depends on the volume of E.

$$\iint_{\Sigma} (3x\mathbf{i} - (ye^z + \sin(x^2))\mathbf{j} + e^z\mathbf{k}) \cdot \mathbf{n}\; dS$$

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11. (10 bonus points) Prove that there is no inner product on $\mathbb{R}^2$ that gives us the norm $||(x_1, x_2)|| = \max\{|x_1|, |x_2|\}$.

(For this problem, partial credit will only be awarded when significant progress towards a solution is made.)