

# Discretely Constrained Mixed Complementary Problems

## :Application and Analysis of a Stylized Electricity Market

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### ABSTRACT

Game theoretic applications with discrete constraints can be found in many areas of engineering and economics. Recent research provides various methods to formulate and solve discretely constrained mixed complementary problems (DC-MCP) by relaxing complementarity. Care must be taken when the DC-MCP is derived from taking the optimality conditions of the continuous version of several optimization problems (e.g., faced by multiple players), concatenating them in to an MCP, and then adding back integer constraints. It may be the case that a DC-MCP solution obtained in this manner does not directly relate to a discretely constrained equilibrium faced by these players.

This paper provides insight into different areas of DC-MCPs. First, we look at three different solution-methods for DC-MCPs from the literature and compare them in terms of solutions and usability. The methods discussed in this paper use disjunctive constraints, special ordered sets of type 1 (SOS1) and an implementation of a certain median function. The different methods are applied to a stylized electricity market including a minimum-generation constraint. The minimum-generation constraint includes binary variables, making the problem a DC-MCP.

Furthermore, the paper discusses the mathematical and economic implications of solutions. It is shown that a relaxed version of the discrete restrictions on variables combined with MCP conditions may not lead to an equilibrium where no player has a unilateral incentive to deviate. To overcome this problem, this paper presents a method to implement a two-stage DC-MCP, using an upper-level service operator to find solutions which are in line with the economic definition of an equilibrium.

### KEYWORDS

DC-MCP; Unit Commitment; Nash-Equilibrium; Welfare-Maximization